Financial Frictions, Liquidity and the Business Cycle

1) (40 points) Consider the following variant of the Diamond and Dybvig (1983) model. The economy lasts for three periods, $t = 0, 1, 2$. There are two consumption goods that give utility in periods 1 and 2. One good is called tradable and the other, non-tradable. There is no endowment of none of the goods in this economy at any date. In period 0 there can only be tradables, which the economy can get from the rest of the world.

There is a continuum of ex-ante identical consumers, with unit mass. As it is standard in this literature, at the beginning of date 1 there exists a preference shock that determines the ex-post type of each consumer. With probability p (the probability is known at date 0) the consumer becomes impatient and with the remaining probability she becomes patient. An impatient agent has preferences represented by the utility function $u(c_{1T}) + v(c_{1N})$, where c_{1l} is the consumption by an impatient consumer (in period 1) of good $l = T, N$, where T stands for tradables and N for non-tradables. A patient consumer has utility function $u(c_{2T}) + v(c_{2N})$, where c_{2l} stands for the consumption at date 2 by the patient agent of good l. Both $u(\cdot)$ and $v(\cdot)$ are increasing and concave. Hence, the ex-ante utility function is:

$$p[u(c_{1T}) + v(c_{1N})] + (1 - p)[u(c_{2T}) + v(c_{2N})] \tag{1}$$

Non-tradables are produced with a constant-returns-to-scale technology. Each unit of tradable invested at date 0 gives either $A > 1$ units of non tradables at date 1, or A^2 units of non-tradables at date 2. It is not possible to invest at date 1 new tradables to produce non tradables. (Thus this linear technology can be seen as allowing reinvestment at date 1 but only of date 0 investments) It is not possible to transform non-tradables back into tradables.

There exist two other investment technologies, standard in the literature. There is a long term investment project that gives $R > 1$ units of tradable goods at date 2 per unit of the T good invested in period 0. As in the literature, assume that if liquidated at date 1 the gross return in terms of tradables is $r < 1$. The other investment corresponds to the fact that the tradable good is assumed to be storable with net return equal to 0.

First, we want to characterize the planner's problem, assuming the existence of credit from the rest of the world that allows the planner to borrow tradable goods directly in the period 0, at a net interest rate equal to 0 (we assume it is not possible to borrow from the rest of the world at date 1). This planner allocates this amount in the different available technologies. Let d be the amount of tradables borrowed by the planner at date 0, let x be the amount of tradables invested in the long term project, and y be the amount of tradables stored between periods 0 and 1. Let z be the amount of tradables invested in 0 to produce non-tradables, and let α be the proportion of z withdraw at period 1 ($0 \leq \alpha \leq 1$). Assume that debt borrowed by the planner at date 0 is repaid in full at date 2.

a) Write the constraints of the planner's maximization. Hint: there are 5, one budget constraint for the planner at date 0, and each one of the remaining four account for the expected consumptions of each type of good for every kind of consumer (of T and N, by

patient and impatient). Write the planner's objective function (1) replacing c_{tl} by the expressions found from the budget constraints.

b) Let z be fixed. Now rewrite the problem in terms of how much of z must be liquidated early (α). Show that the optimal amount of investment liquidated at date 1 in the non-tradables technology is such that $v'(c_{1N}) = Av'(c_{2N})$. Interpret.

c) Write down the other first order conditions for an interior solution (besides the constraints), and show that $c_{1T} < c_{2T}$ and $c_{1N} < c_{2N}$. Explain.

Now we want to study the decentralized outcome without a planner. Assume that debt must be repaid, such that if there is a situation of distress debt is repaid before consumption is determined.

d) Can these optimal assignments be reached by a bank offering deposit contracts? If you can't find a formal proof, give intuition.

e) Find conditions for a bank run to be a Nash equilibrium. Mention (and briefly describe) three ways of resolving that.

2) (60 points) Consider as given the following summary stylized facts of the Danish mortgage market (i.e. even if you know them to be a simplification or incorrect, take them to be true).

Borrowers can freely refinance their mortgages. This is done by paying loans early and simultaneously taking a new loan. This prepayment option exposes lenders to risk. To deal with this risk, mortgages are financed using bonds whose maturity and cash-flow exactly match those of the underlying loans (of course this is done for a large number of loans). These bonds are callable, to reflect repayment risk of underlying mortgages. Assume mortgage issuers are not responsible for making payments on the bonds they issue.

By offering standardized lending contracts, the mortgage-backed securities issued are standardized themselves. This makes mortgage bonds to be very liquid, and thus attractive to investors. Investors are, in principle, Danish pension funds and insurers. But we are going to consider a situation in which the volume of mortgage bonds is so large (because of a boom in property prices) that some of them have to be bought by other investors, we denote the bonds bought by other investors as *excess bonds*. One possibility is that these are Danish banks that must increase their exposure to mortgage risk (buying a pool of diversified bonds issued by other Danish banks). The second possibility is that these buyers are European, non-Danish, banks.

Suppose that the relevant banking regulator perceives an increase in the default risk of Danish mortgages (perhaps due to "scary bad" news that in April 2014 the Danish government passed a law that would automatically extend by a year the maturities of existing bonds in the event that new auctions of mortgage bonds fail, or if interest rates go up by more than 5 percentage points). As a response to this news, the regulator stops considering Danish mortgage bonds as assets that banks can use to meet liquidity requirements.

a) Consider the likely developments in the Danish banking system, and the real economy, in the case in which excess bonds are only held by Danish banks and the regulator is the Danish Central Bank.

b) Consider the likely developments in the Danish banking system, and the real economy, in the case in which excess bonds are only held by European Union, non-Danish, banks and the regulator is the European Central Bank or Banking Authority.

c) How would your answers in a) and b) change if mortgage issuers are responsible for making payments on the bonds they issue?

d) Give reasons why banks have to meet liquidity requirements. Explain how bank liquidity might affect the real economy.

e) Do you think that the decline in Danish property prices from mid 2007 to mid 2009 had any causal effect on economic activity in Denmark? Explain.